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## MATRIX-TOPOLOGICAL MODEL OF ELECTROMAGNETIC CIRCUITS

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**Purpose.** To develop a digital model of electromagnetic devices for research and optimization of powerful secondary electric power sources and electromagnetic converters.

**Methodology.** Nodal potential method, Contour current method, Topologically isomorphic transformations.

**Findings.** The purpose of this work is to create a mathematical apparatus that allows solving problems of modeling and researching electromagnetic devices in parts (by types of accumulated energy). This will simplify the research and optimization of technical characteristics such as efficiency, weight and size indicators, etc. The proposed mathematical model of electromagnetic circuits has the greatest degree of detail of the electric and magnetic circuit. The magnetic circuit is represented in the same detail as the electric circuit, and is described by a contour matrix. A mathematical description of electromagnetic devices is obtained in which inductive parameters are determined by the geometric dimensions and characteristics of magnetic circuits. The topology of the electrical circuit is represented by matrix blocks, which allowed obtaining a mathematical description, which simultaneously takes into account the distribution of currents and charges in the elements of the circuit. The system of equations reduces to the Cauchy form and is composed with respect to increments of magnetic fluxes and potentials on capacitors, which simplifies its solution by numerical methods on a computer. Thus, it is convenient to monitor the energy processes in the reactive power-consuming elements of the circuit. A stable and adaptive digital model of electromagnetic circuits has been developed that makes it possible to combine several methods of integrating a system of differential equations. Feedback is provided through a special parameter. This makes it possible to maximize the correctness of the computations for the energy components in the simulation of the electromagnetic circuit. The originality of the mathematical description lies in the fact that the topology of the electromagnetic circuit is represented in the form of separate matrices that are connected by a matrix of coil connections. The practical value of the digital model of the electromagnetic circuit is that the parameters of the magnetic circuits are introduced in the form of geometric dimensions of the magnetic circuits. This eliminates the need for equivalent transformations to produce data for a specific model. This simplifies the study of secondary power supplies and other powerful electric power consumers by efficiency criteria, weight and size parameters.

**Originality.** The topology of the electromagnetic circuit is represented in the form of separate matrices that are connected by a matrix of coil connections.

**Practical value.** The parameters of the magnetic circuits are introduced in the form of geometric dimensions of the magnetic circuits.

**Keywords:** electric circuit; magnetic circuit; static electromagnetic devices; topological isomorphic modeling; matrix topological description; topological matrices; incidence matrices; block structure of the topological matrix; matrix of helical links; secondary power supplies; pulse current generators; topological isomorphic model.

### I. INTRODUCTION

Modern computer-aided design systems require the development of special mathematical software, which will provide the most simulation for the devices being developed. The main requirements for the development of models can be the greatest degree of detail, the permissible quality of modeling, the simplicity of obtaining model parameters.

A special role is played by design automation in the electric power industry [6], [11]. In autonomous electric power systems, all devices can be divided into three groups: power supplies, converters (or secondary power supplies), and electricity consumers.

Among them, devices that belong to the second group, by mass and size, are sometimes commensurate with power supplies and often exceed by these parameters consumers of electricity. In addition, electricity converters

are also a kind of energy consumers, which is used to control switching elements and is released as heat. The amount of energy consumed by the converter affects the economics of the autonomous system for the worse, and is therefore often the criterion for research.

Today, computer modeling of electrical and electronic devices continues to play a special role in developing economically and technically efficient devices for various purposes. This allows not only to speed up the calculation work and improve the design quality, but also to create favorable conditions for the development of the theory of modeling energy-intensive parts of electric power converters.

The complexity of studying nonlinear electric and magnetic circuits is that there are no general analytical solutions of systems of nonlinear algebraic and differential equations. It is generally believed that the analysis of essentially non-linear chains, with the exception of a

small number of simplest cases, can be performed only numerically.

This material was compiled from the results of [12], [20] – [21].

## II. ANALYSIS OF LAST RESEARCHES

When modeling devices, it is very important to choose a method that allows you to accurately estimate the accumulation and loss of energy in the elements of the circuit. This may affect the final result of the study, for example, the efficiency, the power factor, and the like.

In the study of devices using computers, the principles of structure representation and methods for describing electromagnetic circuits (EMC) play an important role. The principles of topological-isomorphic modeling (TIM) are well studied and successfully used in the automation of EMC design [12]. However, modern electronic devices with electromagnetic components in their composition usually have a large number of capacitive elements, which requires the further development of TIM methods and methods of EMC description. One way to solve this problem is to divide the EMC into subcircuits.

Choosing the appropriate description for the study of physical processes, you should take into account the purpose, direction of development, the degree of detail of the device nodes, are modeled. One of the most important problems of automation of scientific research of electromagnetic devices (EMD) is the study of energy processes and characteristics (efficiency, power factor, etc.). In this case, it is necessary to take into account the energy losses at each element of the circuit. A number of devices (electromagnetic converters, electric and magnetic energy storage devices, power supplies of electronic equipment, etc.) include powerful magnetic systems where there are losses to magnetic hysteresis and eddy currents. To take into account the losses in steel, it is necessary to represent the magnetic circuit in the same detail as the electric circuit.

At present, methods for representing EMC have been developed, where the separation of the circuit into electric and magnetic circuits is used. To represent the graph of a magnetic circuit, use an incidence matrix or, more often, a contour matrix of magnetic bonds. The connection between the magnetic and electric graphs is provided by the matrix  $W$ , the dimension of which depends on the number of turns located on the magnetic rods and the method of connections [12]. There are two ways of forming  $W$ . In one case, the connection of the electric branch to the magnetic flux circuit is realized, in the other case the connection of the contour stream to the electric branch is realized. The choice of the matrix is determined by the concrete implementation of the mathematical models of the EMC.

The structure of electrical circuits is described by matrices of incidents, contour currents, cross sections, etc., makes it possible to widely use the theory of matrices in the compilation of EMC descriptions. In turn, the the-

ory of matrices allows solving the problems of modeling electric and magnetic circuits by parts [5], [7], [13], [16] – [17], [23]. If all known types of elements (resistors, capacitors, inductors, etc.) are present in the circuits, then the matrices that retain the electrical circuit structure do not agree well with the mathematical description. In such cases, the electrical circuit is divided into subcircuits in which elements of the same type are localized. For example, it is possible to isolate the linear resistive part of the circuit, and the nonlinear elements and components accumulating energy are replaced by current sources and electromotive forces. Thus, the designer of EMD must deal with the transformation of the original and the construction of a new (equivalent) scheme.

## III. FORMULATION OF THE WORK PURPOSE

In view of the above, it is possible to formulate the goal of the work as follows: to develop a digital model of electromagnetic circuits that is convenient for researching and optimizing powerful secondary power supplies and electromagnetic converters. In this model there should be the following:

- separation of the elements of the scheme according to the types of accumulated energy of the magnetic and electric fields and the consumed energy radiated in the form of heat;
- representation of the magnetic components of the circuit in the form of their design parameters and the magnetic properties of the material used;
- eliminating the need for an equivalent transformation scheme;
- adaptability to the use of application packages, which by their organization use a matrix representation of data (for example, MATLAB).

## IV. DEVELOPMENT OF THE MATRIX-TOPOLOGICAL MODEL OF ELECTROMAGNETIC CIRCUITS

The review of the diagrams of the power part of the EMU proposed in the theoretical and reference literature [1] – [2], [8], [10], [19], [24], etc. makes it possible to use the following fragment of an electrical circuit, on the basis of which a mathematical model will be developed (see Fig.1).

Limitations on the number of branches and the complexity of magnetic structures are not superimposed. Exceptions are schemes with multiple multiplication of voltage (for example, the Latour scheme [12]), unified circuits for high-voltage transformer-rectifier modules [10] and some others, the complexity of which lies in the fact that in them capacitive couplings can not be represented as a "star" (see Fig.2).

In these cases, it is necessary to solve the problem of charge distribution between capacitors or to transform the circuit by introducing an active resistance into some capacitive branches.

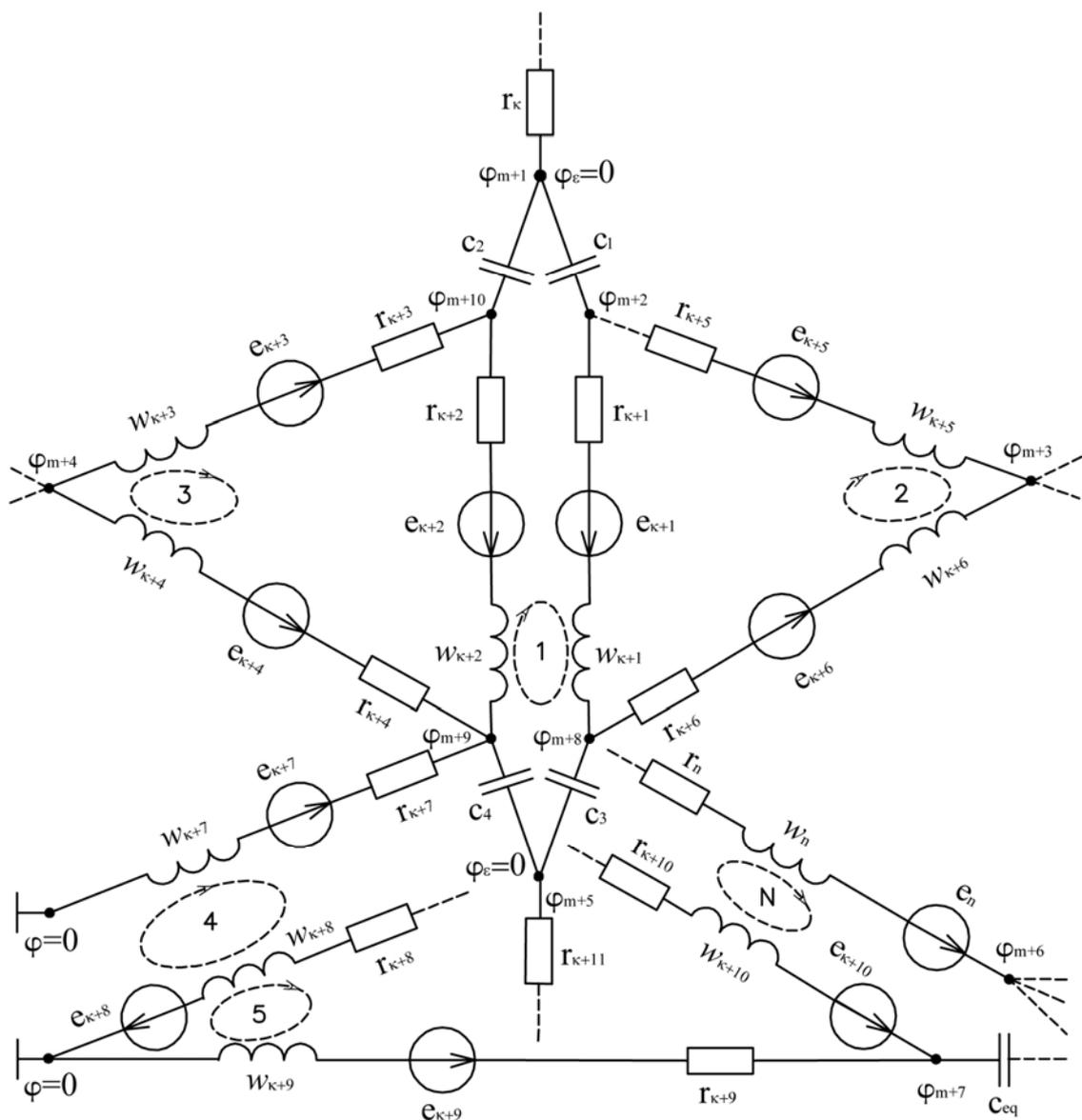


Figure 1. Fragment of the electrical circuit of a static electromagnetic device

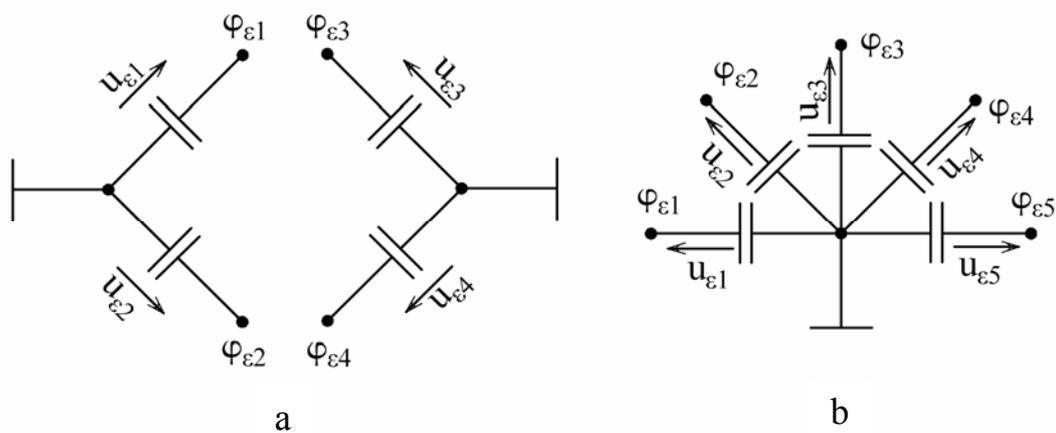


Figure 2. Capacitive subcircuits

Simulation of electrical circuits is greatly simplified, if it can be solved with respect to electrical voltages on capacitors. This follows from the fact that the number of capacitive elements, as a rule, does not exceed 40-50% of the number of components in the circuit. Therefore, the problem of numerical modeling is simplified in the sense of reducing the number of unknown variables.

Let the electrical circuits for which a mathematical model is developed are represented with two-pole components.

Then, depending on the nature of the components, the branches of the graph of electrical circuits are divided into the following types:

- 1) branches of passive two-ports (capacitances);
- 2) branches of dependent and independent voltage sources with linear and nonlinear active resistances.

Considering branches by types, the graph of the electrical circuit can be conditionally divided into subgraphs: "capacitive" and "resistive". In the future, the concept of "subgraph" will be replaced by a "graph" when we consider them as separate structures. If you first number the branches and nodes of the arcs of the capacitive graph, and then the branches and nodes of the resistive graph, then the incidence matrix, which reflects the structure of the electrical circuit, is automatically divided into blocks:

$$A = \begin{vmatrix} A_{\varepsilon} & A_{\varepsilon r} \\ A_o & A_r \end{vmatrix},$$

where matrix blocks are:

$A_{\varepsilon}$  – incidence matrix of incoming branches of the graph of the capacitive part of the electronic circuit;

$A_o$  – incidence matrix of outgoing branches of the graph of the capacitive part of the electronic circuit;

$A_{cr}$  – incident matrix, where the resistive branches are incident with the capacitive;

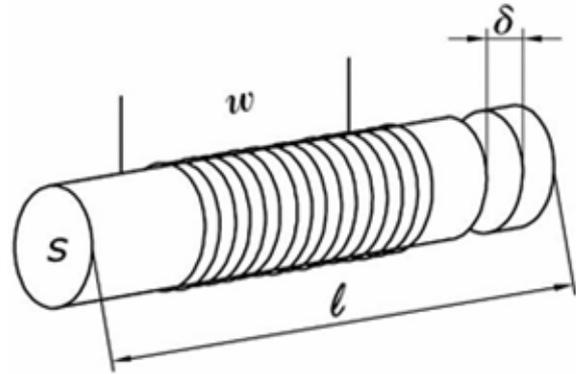
$A_r$  – incident matrix, where the resistive branches are not incident with capacitive branches.

Thus, the "capacitive" graph is described by a topological submatrix  $A_c$ . The basic nodes of it are placed in a separate block  $A_o$ . This block provides a connection between the base nodes of the "capacitive" graph and the "resistive" graph, which is represented by the submatrix  $A_r$ . Other nodes of the capacitive graph can be and are most often associated with a "resistive" graph, and this relationship is described by a submatrix  $A_{\varepsilon r}$ .

#### Description of the structure of the magnetic circuit and its connection with the electrical circuit

The magnetic circuit is represented in as much detail as the electric circuit and is described by a contour matrix  $\Gamma_m$ . In order to describe scattering fluxes, when forming a graph, each magnetic connected chain can be supplemented by an "air" branch [12].

The parameters of the magnetic branch can be calculated as follows (see Fig. 3).



**Figure 3.** Constructive parameters of the magnetic branch

The rod made of magnetic material (length -  $\ell$ , cross-section -  $s$ , gap -  $\delta$ ) is wound with one or several windings with a number of turns  $w$ . The magnetic resistance of the branch is calculated by the formula

$$r_m = \frac{\ell - \delta}{s\mu} + \frac{\delta}{s\mu_0},$$

where  $\mu$  - magnetic permeability of a rod,  $\mu_0$  - magnetic permeability of a gap.

The cross-section of the gap and the rod is assumed to be the same under the condition that  $\delta \ll \ell$ . The inductance of the branch can be determined by the expression:

$$L = \frac{w^2}{r_m}.$$

The connection between the magnetic and electric graphs is provided by the matrix  $W$ . Its dimension in rows corresponds to the contour matrix  $\Gamma_m$ , and by the columns - the conductivity matrix  $Y$ . Elements of the matrix are filled in the following way. If the beginning and end of the electrical winding coincide with the chosen direction of the magnetic flux, then the number of turns with a plus sign is inscribed in the corresponding cell, if the directions do not coincide, then the number of turns with a minus sign.

#### Topological-isomorphic description of the electro-magnetic circuit

Consider the electrical circuit of the static electro-magnetic device in Fig. 1. The numbering of branches and knots is performed according to the above rules for forming a matrix  $A$ . From the general scheme, a capacitive subscheme can be distinguished (see Fig. 2, a).

In what follows we confine ourselves to the consideration of capacitive subgraphs of the "star" type. For graphs such as "triangle" you need a special algorithm for the redistribution of charges. After dividing the circuit into resistive and capacitor nodes, we turn to the problem

of the distribution of currents between the branches of both subgraphs.

The instantaneous stresses of the branches of the electric graph are determined by the potentials of the nodes and the voltages on the capacitors. Let the node  $\varphi_{m+2}$  (see Fig. 1) is not an incident capacitor and it is necessary to find the voltage drop on the branch  $r_{k+5} - e_{k+5} - w_{k+5}$ . The sought voltage is determined by the potential difference  $\varphi_{m+2}$  and  $\varphi_{m+3}$ . The algorithmic complexity consists in determining the node's potential, if the capacitor is incident with it. Define it as:

$$\varphi_{m+2} = \varphi_{m+1} + u_{c1},$$

where  $u_{c1} = -\varphi_{c1}$  - voltage on the capacitor (see Fig. 2, a).

Then

$$\varphi_{m+2} = \varphi_{m+1} - \varphi_{c1}.$$

Using the block structure of the matrix  $A$ , determine the distribution of currents in the circuit as follows:

$$I = Y \left[ A_r^t V_r + A_{er}^t (V_\varepsilon - A_\varepsilon A_0^t V_r) + E - W^t \Phi' \right], \quad (1)$$

where  $I, E, \Phi$  - respectively, currents, electromotive forces and magnetic fluxes, and the component  $W^t \Phi'$  takes into account the voltage drop across the magnetic coils.

Expression (1) implements the method of nodal potentials using separate matrix blocks  $A$ :  $A_\varepsilon, A_{er}, A_0, A_r$ .

The relationship between the magnetic flux and the current of the electrical circuit is determined as follows:

$$WI = \Gamma_m R_m \Gamma_m^t \Phi, \quad (2)$$

where  $R_m$  - diagonal matrix of magnetic resistances.

The derivative of the voltage on the capacitor is determined by its charge current:

$$A_\varepsilon C A_\varepsilon^t V'_\varepsilon = -A_{er} I. \quad (3)$$

We draw a section covering both nodes to which the capacitor is connected. Then the sum of the currents flowing in the cut branches according to the first Kirgoth law will be zero. Using the topological matrix  $A$ , this law is realized as follows:

$$A_r I - A_0 A_\varepsilon^t A_{er} I = 0. \quad (4)$$

The above relations relate all the main variables that determine the state of the electric and magnetic circuit. Thus, the matrix equations (1) - (4) form a complete system of differential equations of the EMC.

The matrix form of the representation of the electromagnetic system is ideally suited to the realization of a

model on electronic computers using application software packages, for example, MATLAB, in which all objects of computation are represented by matrices [19].

The mathematical description (1) - (4) provides for the use of resistors, capacitances, inductive elements and EMF sources. However, in the converter devices there are elements with nonlinear characteristics - controlled and uncontrolled semiconductor diodes, transistors, cores of magnetic material and the like. To model the converter devices with these elements, without going beyond the scope of the description (1) - (4), requires the use of special mathematical models of the above elements, which must be added to this model in digital implementation on a computer. In this connection, only numerical methods can be used to solve system (1) - (4). The abundance of state variables in it:  $I, \Phi, \Phi', V_\varepsilon, V'_\varepsilon, V_r$ , - complicates the solution, in addition, requires the use of special iterative methods.

Iterative methods for solving systems of differential equations are well studied in [20], [22] and are successfully used in numerical study of nonlinear systems [20]. Attempts have been made to develop original simple fast-acting methods [22], but the problems of the dimension of problems, algorithmic difficulties in the iteration step and their number remain.

In our case, the use of iterative methods of investigation would not be effective, since the task posed - energy research, in particular taking into account dispersion and accumulation of electrical energy on each element of the scheme - will require additional computational costs of the computer time [20].

In addition, modeling of nonlinear magnetic characteristics of materials requires the development of special models that should be included in the general algorithm for the study of chains.

On the basis of the foregoing, machine time must be saved, and this is achieved by using explicit methods of numerical integration, the main advantages of which are described in [9].

### Reduction of the topologically isomorphic description of the EMC to a form convenient for solving by numerical methods

When developing a mathematical model on a computer using explicit methods of numerical integration, it is necessary to solve the system (1) - (4) with respect to the derivatives of the magnetic flux vectors  $\Phi'$  and the capacitor voltages  $V'_\varepsilon$ .

To do this, we perform matrix transformations as follows. We re-group the terms in (1) as follows:

$$I = Y \left[ \left( A_r^t - A_{er}^t A_\varepsilon A_0^t \right) V_r + A_{er}^t V_\varepsilon + E - W^t \Phi' \right] \quad (5)$$

We introduce a new topological matrix:

$$A_r^t = A_r^t - A_{er}^t A_\varepsilon A_0^t,$$

then

$$A_\gamma = A_r - A_0 A_\varepsilon^t A_{\varepsilon r}.$$

If, we substitute expression (5) in (4), then we get:

$$A_\gamma Y A_\gamma^t + A_\gamma Y (A_{\varepsilon r}^t V_\varepsilon + E - W^t \Phi') = 0.$$

From here you can get  $V_r$ , to exclude it from equation (5):

$$V_r = [A_\gamma Y A_\gamma^t]^{-1} A_\gamma Y (W^t \Phi' - A_{\varepsilon r}^t V_\varepsilon - E).$$

Eliminate the vector  $V_r$  from equation (5) and obtain:

$$I = Y A_\gamma^t [A_\gamma Y A_\gamma^t]^{-1} A_\gamma Y (W^t \Phi' - A_{\varepsilon r}^t V_\varepsilon - E) - Y (W^t \Phi' - A_{\varepsilon r}^t V_\varepsilon - E) \quad (6)$$

We introduce the matrix of active parameters:

$$Y_r = Y \left[ \varepsilon - A_\gamma^t [A_\gamma Y A_\gamma^t]^{-1} A_\gamma Y \right],$$

where  $\varepsilon$  - unit matrix.

Then equation (6) takes the form:

$$I = Y_r (A_{\varepsilon r}^t V_\varepsilon + E - W^t \Phi'). \quad (7)$$

Thus, we obtained an equation in the form of nodal potentials, where all the potentials of the electric circuit graph are determined through potentials on capacitances in a capacitive subgraph.

Eliminate the vector  $I$  in the description (1) - (4) by substituting (7) into (2) and (3), which gives:

$$W Y_r (A_{\varepsilon r}^t V_\varepsilon + E - W^t \Phi') = \Gamma_m R_m \Gamma_m^t \Phi.$$

From this we determine the derivative of the magnetic flux:

$$\Phi' = [W Y_r W^t]^{-1} [W Y_r (A_{\varepsilon r}^t V_\varepsilon + E - F)], \quad (8)$$

where

$$F = \Gamma_m R_m \Gamma_m^t \Phi.$$

Vector  $F$  can be defined and thus:

$$F = \Gamma_m L H,$$

where  $L$  - matrix of lengths of magnetic cores;  $H$  - vector-column of magnetic tension in rods.

We substitute equation (7) in (3):

$$A_\varepsilon C A_\varepsilon^t V_\varepsilon' = -A_{\varepsilon r} V_r (A_{\varepsilon r}^t V_\varepsilon + E - W^t \Phi').$$

Let us single out the derivative of the electric potential vector on capacitors:

$$V_\varepsilon' = [A_\varepsilon C A_\varepsilon^t]^{-1} A_{\varepsilon r} V_r (W^t \Phi' - A_{\varepsilon r}^t V_\varepsilon - E). \quad (9)$$

Thus, the system (1) - (4) is solved with respect to the derived vectors  $\Phi'$  and  $V_\varepsilon'$ . System (8) - (9) is a description of the electromagnetic circuit, resolved with respect to magnetic fluxes and electrical potentials on the capacitors. A feature of this description is that it allows you to model ideal electromagnetic circuits, since it does not have redundancy. Other special cases of calculating such chains are known, for example, in [14]. In them, ideal transformers are considered as additional structural equations. However, this leads to a complication of the mathematical model. In our case, the dimension of the problem is significantly reduced. The system (8) - (9) is written in the Cauchy form and is suitable for investigation by explicit methods of numerical integration.

### The problem of rigidity

As the simulation results show, the mathematical model of the EMC (8) - (9) behaved well when calculating the transformer circuit [20]. However, in the general case there can be problems associated with the rigidity of the equations and, in connection with this, with the control of the computational process.

In the general case, the components of the parameters  $Y, C, R_m$  of electrotechnical devices can differ greatly from each other. This leads to the rigidity of systems of differential equations. The concept of rigid equations was first introduced in 1952 by K. Curtis and D. Hirschfelder [21]. A feature of such systems is the need to reduce the iteration step when using the classical methods of Euler, Runge-Kutta, Adams, and others:

$$h < \tau_{b.l.}, \quad (10)$$

where  $h$  - integration step;  $\tau_{b.l.} < b - a$  - boundary layer [15];  $[a, b]$  - a segment of observation (may correspond to the period of the fundamental harmonic of the process under study).

The fulfillment of condition (10) allows to display high-frequency components, which are a consequence of physical processes in charge circuits with small time constants and in high-frequency oscillatory circuits.

However, the

$$N = \frac{T}{\tau_{n.c.}},$$

where  $T$  - the period of the fundamental harmonic component of the process under investigation may turn out to be such that the use of standard methods will be unsuitable because of the long calculation time.

In this case, inverse methods of numerical integration are useful: the inverse Euler method [21], the trapezium method, the differentiation formulas back [21], etc. These methods are good in that, depending on the step of numerical integration, we get the corresponding "detail" of the physical process, while the stability of the computational process is not violated.

It is known that the errors of the explicit and implicit

methods are close in modulus and are proportional  $h^2$ , but are different in sign [21]. Therefore, combining these methods, you can get a good approximation. This is achieved by calculating the derivatives of state vectors with respect to different points within the integration interval [21].

If the linear approximation is performed at the integration step, then increments  $\Delta\Phi$  and  $\Delta V_\varepsilon$  can be determined by solving a system of algebraic equations. We will show this with an example. Let

$$X = \begin{bmatrix} \Phi \\ V_\varepsilon \end{bmatrix},$$

then

$$AX' = BX + CE, \quad (11)$$

where  $A, B, C$  - square matrices;  $X, E$  - vector columns.

We write the system (11) in the following form:

$$X' = A^{-1}(BX + CE). \quad (12)$$

The increment at the integration step with respect to different points within the interval

$$\Delta X = h \left[ A^{-1}B \left( X + \frac{\Delta X}{z} \right) + A^{-1}CE \right]. \quad (13)$$

We solve system (6) with respect to  $\Delta X$ :

$$\Delta X = \left( I - \frac{hA^{-1}B}{z} \right) hA^{-1}(BX + CE). \quad (14)$$

We represent the derivatives (8) - (9) inside the interval:

$$\Phi' = F_\Phi \left( \Phi + \frac{\Delta\Phi}{z}, V_\varepsilon + \frac{\Delta V_\varepsilon}{z}, E + \frac{\Delta E}{z} \right) \quad (15)$$

$$V_\varepsilon' = F_V \left( \Phi + \frac{\Delta\Phi}{z}, V_\varepsilon + \frac{\Delta V_\varepsilon}{z}, E + \frac{\Delta E}{z} \right) \quad (16)$$

If you perform operations similar to (12) - (14) over system (8) - (9), you can get:

$$\begin{bmatrix} h^{-1}WY_\rho W^t + z^{-1}\Gamma_m R_m \Gamma_m^t \\ -h^{-1}A_{\varepsilon\rho} Y_\rho W^t \\ -z^{-1}WY_\rho A_{\varepsilon\rho}^t \\ h^{-1}A_\varepsilon C A_\varepsilon^t + z^{-1}A_{\varepsilon\rho} Y_\rho A_{\varepsilon\rho}^t \end{bmatrix} \times \begin{bmatrix} \Delta\Phi \\ \Delta V_\varepsilon \end{bmatrix} = \begin{bmatrix} WY_\rho (A_{\varepsilon\rho}^t V_\varepsilon + E) - F(\Phi) \\ -A_{\varepsilon\rho} Y_\rho (A_{\varepsilon\rho}^t V_\varepsilon + E) \end{bmatrix}. \quad (17)$$

Relatively  $\Delta\Phi$  and  $\Delta V_\varepsilon$  system (17) can be solved by any numerical method. Varying the value  $z$ , we can

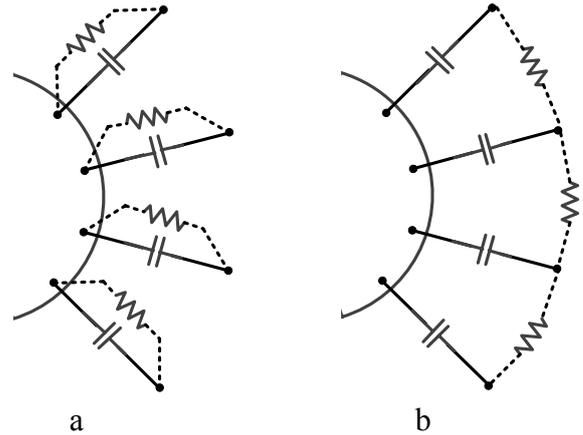
change the methods of numerical integration, for example:

- a) at  $z = \infty$  - explicit Eulerian method;
- b) at  $z = 1$  - inverse Euler method;
- c) at  $z = 2$  - trapezium method and others.

Parameter  $z$  can also take any non-integer value.

The square matrix in system (17) can be poorly conditioned in one case, when there are capacitors with "hanging" nodes in the circuit. This may occur in the study of chains in parts. In this case, it is recommended to shunt the "hanging" capacitors with resistors in one of the above ways (see Fig. 4). The variant (Fig. 4, b) is preferable, since it requires fewer additional elements.

Thus, a controlled digital EMC model (17) has been developed that satisfies the requirements of stability under stiffness conditions for systems of differential equations. Obviously, it will be necessary to define a criterion for optimizing a numerical solution that allows us to control the computational process by means of  $z$ .



**Figure 4.** Methods for improving the conditionality of the matrix (17) by shunting with resistors of "hanging" capacitances

#### IV. CONCLUSION

1. The topology of the electrical circuit is represented by matrix blocks  $A_\varepsilon$ ,  $A_{\varepsilon\rho}$ ,  $A_o$ ,  $A_r$ , which made it possible to obtain a mathematical description that simultaneously takes into account the distribution of currents and charges in the elements of the circuit.

2. A mathematical description of electromagnetic devices (1) - (4) is obtained, in which inductive parameters are determined by the geometric dimensions and characteristics of the magnetic cores.

3. The matrix form of the representation of the electromagnetic system ideally approaches the implementation of the model on electronic computers using application software packages, for example, MATLAB, in which all the calculation objects are represented by matrices.

4. Proceeding from the given structure of the mathematical description of the EMC, it is expedient to use the proposed model for the study of devices that have magnetic cores in their composition that are powerful enough to account for them. This model was used to investigate secondary power supplies and powerful current generators.

5. In this paper, special attention is paid to the simplicity of presenting the initial information, excluding equivalent transformations of the electrical circuit.

6. A stable mathematical model of electromagnetic circuits in the matrix form (17) is developed, which is convenient for realization on digital computers. The model is composed relative to the increments of magnetic fluxes and potentials on capacitors.

7. Computational methods are controlled by the parameter  $z$ .

The developed software is well-algorithmized on digital computers and can be used to create specialized software systems.

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## МАТРИЧНО-ТОПОЛОГІЧНА МОДЕЛЬ ЕЛЕКТРОМАГНІТНИХ КІЛ

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**Мета роботи.** Розробити цифрову модель електромагнітних пристроїв для дослідження та оптимізації потужних вторинних джерел живлення та електромагнітних перетворювачів.

**Методи дослідження.** Метод вузлових потенціалів, метод контурних струмів, топологічно-ізоморфні перетворення.

**Отримані результати.** Сучасні системи автоматизованого проектування вимагають розробки спеціального математичного забезпечення. Основними вимогами до розробки моделей можуть бути найбільша ступінь деталізації, допустима якість моделювання, простота отримання параметрів моделі. В автономних електроенергетичних системах всі пристрої можна поділити на три групи: джерела живлення, перетворювачі і споживачі електроенергії. Серед них пристрої, що відносяться до другої групи, за масою і габаритами іноді сумірні з джерелами живлення і часто перевищують за цими параметрами споживачі електроенергії. Кількість спожитої енергії перетворювачем впливає в грішу сторону на економічність автономної системи і тому часто є критерієм дослідження. Метою даної роботи є створення математичного апарату, що дозволяє вирішувати задачі моделювання та дослідження електромагнітних пристроїв по частинах (за видами накопичуваної енергії). Це дозволить спростити дослідження і оптимізацію таких технічних характеристик, як коефіцієнт корисної дії, масогабаритні показники тощо. Запропонована математична модель електромагнітних кіл має найбільший ступінь деталізації електричного і магнітного кола. Магнітне коло представлено так само докладно, що і електричне коло, і описується контурною матрицею. Отримано математичний опис електромагнітних пристроїв, в якому індуктивні параметри визначаються геометричними розмірами і характеристиками магнітного кола. Топологія електричного кола представлена матричними блоками, що дозволило отримати математичний опис, котрий одночасно враховує розподіл струмів і зарядів в елементах схеми. Система рівнянь зводиться до форми Коші і складена відносно проросту магнітних потоків і потенціалів на конденсаторах, що спрощує її рішення чисельними методами на комп'ютері. Таким чином, зручно стежити за енергетичними процесами в реактивних енергоємних елементах схеми. Розроблена стійка і адаптивна цифрова модель електромагнітних кіл, що дозволяє об'єднати кілька методів інтегрування системи диференціальних рівнянь. Зворотній зв'язок надається через спеціальний параметр. Це дозволяє домогтися максимальної коректності обчислень для енергетичних компонентів при моделюванні електромагнітного кола. Оригінальність математичного опису полягає в тому, що топологія електромагнітного кола представлена у вигляді окремих матриць, котрі пов'язані між собою матрицею виткових зачеплень. Практична цінність цифрової моделі електромагнітного кола полягає в тому, що параметри магнітних кіл вводяться в вигляді геометричних розмірів магнітопроводів. Це виключає необхідність проводити еквівалентні перетворення для підготовки даних конкретної моделі. Це спрощує вивчення вторинних джерел живлення та інших потужних споживачів електроенергії за критеріями ефективності, вагових і розмірних параметрів.

**Наукова новизна.** Топологія електромагнітного кола представлена у вигляді окремих матриць, з'єднаних матрицею виткових зачеплень.

**Практична цінність.** Параметри магнітних кіл вводяться у вигляді геометричних розмірів магнітних кіл.

**Ключові слова:** статичні електромагнітні пристрої; топологічно-ізоморфне моделювання; матричний топологічний опис; топологічні матриці; матриці інцидентів; блокова структура топологічної матриці; матриця виткових зачеплень; вторинні джерела живлення; генератори імпульсних струмів.

## МАТРИЧНО-ТОПОЛОГИЧЕСКАЯ МОДЕЛЬ ЭЛЕКТРОМАГНИТНЫХ ЦЕПЕЙ

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**Цель работы.** Разработать цифровую модель электромагнитных устройств для исследования и оптимизации мощных вторичных источников питания и электромагнитных преобразователей.

**Методы исследования.** Метод узловых потенциалов, метод контурных токов, топологически-изоморфные преобразования.

**Полученные результаты.** Современные системы автоматизированного проектирования требуют разработки специального математического обеспечения. Основными требованиями к разработке моделей могут быть наибольшая степень детализации, допустимое качество моделирования, простота получения параметров модели. В автономных электроэнергетических системах все устройства можно подразделить на три группы: источники питания, преобразователи и потребители электроэнергии. Среди них устройства, относящиеся ко второй группе, по массе и габаритам иногда соизмеримы с источниками питания и часто превышают по этим параметрам потребители электроэнергии. К тому же преобразователи электроэнергии также является своего рода потребителями энергии, которая используется для управления коммутационными элементами и выделяется в виде тепла. Количество потребленной энергии преобразователем влияет в худшую сторону на экономичность автономной системы и поэтому часто является критерием исследования. Целью данной работы является создание математического аппарата, позволяющего решать задачи моделирования и исследования электромагнитных устройств по частям (по видам накапливаемой энергии). Это позволит упростить исследование и оптимизацию таких технических характеристик, как коэффициент полезного действия, массогабаритные показатели и т.д. Предложенная математическая модель электромагнитных цепей имеет наибольшую степень детализации электрической и магнитной цепи. Магнитная цепь представлена так же подробно, что и электрическая цепь, и описывается контурной матрицей. Получено математическое описание электромагнитных устройств, в которых индуктивные параметры определяются геометрическими размерами и характеристиками магнитных цепей. Топология электрической цепи представлена матричными блоками, что позволило получить математическое описание, которое одновременно учитывает распределение токов и зарядов в элементах схемы. Система уравнений сводится к форме Коши и составлена относительно приращений магнитных потоков и потенциалов на конденсаторах, что упрощает ее решение численными методами на компьютере. Таким образом, удобно следить за энергетическими процессами в реактивных энергоемких элементах схемы. Разработана устойчивая и адаптивная цифровая модель электромагнитных схем, позволяющая объединить несколько методов интегрирования системы дифференциальных уравнений. Обратная связь предоставляется через специальный параметр. Это позволяет добиться максимальной корректности вычислений для энергетических компонентов при моделировании электромагнитной цепи. Оригинальность математического описания заключается в том, что топология электромагнитной цепи представлена в виде отдельных матриц, которые связаны между собой матрицей витковых зацеплений. Практическая ценность цифровой модели электромагнитной цепи заключается в том, параметры магнитных цепей вводятся в виде геометрических размеров магнитопроводов. Это исключает необходимость проводить эквивалентные преобразования для подготовки данных конкретной модели. Это упрощает изучение вторичных источников питания и других мощных потребителей электроэнергии по критериям эффективности, весовых и размерных параметров.

**Научна новизна.** Топология электромагнитной цепи представлена в виде отдельных матриц, соединенных матрицей витковых зацеплений.

**Практическая ценность.** Параметры магнитных цепей вводятся в виде геометрических размеров магнитных цепей.

**Ключевые слова:** статические электромагнитные устройства; топологически-изоморфное моделирование; матрично-топологическое описание; топологические матрицы; матрицы инцидентий; блочная структура топологической матрицы; матрица витковых зацеплений; вторичные источники питания; генераторы импульсных токов.