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# SPECTRAL METHOD OF ELECTRICAL CIRCUITS ACCELERATED SIMULATION WITH THYRISTORS

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**Purpose.** The development of transient processes calculation method in electric circuits with thyristors based on the use of functions approximation by orthogonal polynomials.

**Methodology.** Functions approximation by orthogonal polynomials, numerical methods of differential equations integration, matrix methods, programming, theory of electric circuits.

**Obtained results.** The method of solution function polynomial approximation of integro-differential equations of state, which describes the transient processes of an electric circuit with thyristors, is used in this paper. The used method showed the advantages over other known methods in increasing the accuracy and reducing the simulation time of transient electrical processes by more than 6 times.

**Findings.** The solution is approximated by a series of Chebyshev polynomials. The integro-differential equations of state are transformed into linear algebraic equations for special depiction of the solution functions. The depiction of functions of true currents in the equivalent circuit is interpreted as direct currents. Such a schematic model creates visibility for a researcher performing simulation of transient electrical processes.

**Practical value.** The proposed methods discover the possibility of using the apparatus of direct current electric circuits' theory for transient processes in complex schemes modeling with thyristors.

**Key words:** electric circuits; orthogonal polynomials; differential equations; numerical methods; spectral methods; approximation; Chebyshev polynomials; transient processes; schematic model.

### I. INTRODUCTION

Many papers [1], [2] have been devoted to the problems of complex electrical circuits modeling with thyristors. Back in the 1980s, a number of universal models of converting devices, oriented to ES series PC, were developed. Therefore, the developed models had a very complex algorithm. Modern computers have much better resources, which are preferably to use. Mathematic numerical methods have also been developed at a rapid pace. The method proposed in this paper can significantly increase the simulation speed. The method of differential equations numerical solution based on the use of orthogonal polynomials is used.

### II. ANALYSIS OF RESEARCH AND PUBLICATION

The paper [3] shows the use of orthogonal polynomials for the differential equations integration. The author calls the method of functions decomposition into series by orthogonal polynomials the spectral method in this paper. An analogy of functions decomposition into a Fourier

series by trigonometric functions is used, where the set of decomposition coefficients is a frequency spectrum. The paper presents studies of the convergence rate of series. The author considers the terms "interpolation" and "collocation" to be identical, but the term interpolation is used to represent a known function, and the term "collocation" is used when decomposing a function, obtained as a result of solving a differential equation, into a series. The paper [4] gives the basics of the spectral method using Chebyshev polynomials for the integration of differential equations and shows the advantages of this method over the finite difference method. The paper [5] presents basics of the spectral method. Chebyshev and Legendre polynomials are considered as basic functions. The application of the method to the solution of ordinary differential equations is shown.

In paper [6], the fundamentals of the spectral method for solving equations in partial derivative problems of hydrodynamics are investigated. The better efficiency of this method compared to finite difference methods is noted. Paper [7] also outlines the basics of the spectral

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method for solving partial differential equations. The presentation is performed without unnecessary abstract mathematics and details. The author tries to show that spectral methods do not have great complexity, as some experts believe. The authors of [8] claim that the spectral methods are the best for solving ordinary differential equations with high accuracy, provided that the data defining the task is smooth. At the same time, it is possible to achieve a high accuracy, when the method of finite differences or finite elements gives two or three signs. Spectral methods require less computer memory comparing to alternative methods.

The solution approximation for the current function in an electric circuit by series in Chebyshev polynomials is used in paper [9]. The transient process in the electric circuit is investigated. The transformation of the integro-differential equation of state into an algebraic equation for currents depiction is shown. Kirchhoff's laws for currents depiction are proven. As a result, a method allowing significant increase of simulation speed is obtained.

A study of transient processes calculation methods in linear electric circuits based on the use of orthogonal polynomials is performed in paper [10], [11]. Based on the proposed methods, a program has been developed [12].

A computer program for magnetic electric circuits with thyristors simulation has been developed by authors [13]. The program uses Gere's numerical methods of ordinary differential equations integrating. Nonetheless, computer modeling is slow.

The orthogonal polynomials are successfully used to integrate differential equations. However, there are no publications showing the use of spectral methods for transient processes calculating in electric circuits with thyristors

### III. FORMULATION OF THE WORK PURPOSE

The development of transient processes calculation method in electric circuits with thyristors based on the use of approximation of functions by orthogonal polynomials.

## IV. EXPOUNDING THE MAIN MATERIAL AND RESULTS ANALYSIS

In many cases, with an accuracy sufficient for practical calculations, the valves are modeled as variable resistive elements having a very small given resistance *Ro* in the open state and a very large given resistance *Rz* in the closed state.

The thyristor maintains its open state if the current through the thyristor is greater than zero and changes from open to closed if the current through the thyristor changes sign from positive to negative. If the thyristor is closed, it can be opened under the fulfillment of two conditions simultaneously: the current through the thyristor must be greater than zero and a positive control pulse is applied to the control electrode. Current time since the beginning of the last EMF period is:

$$t_p = t - \left[ \frac{t}{T_i} \right] \cdot T_i , \qquad (1)$$

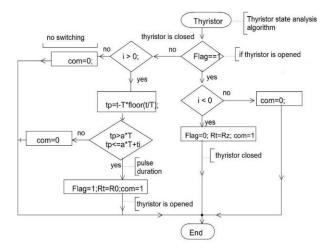
where t is the current time, and square brackets mean the operation of taking an integer part of a number.  $T_i$  is the pulse tracking period on the controlling electrode.

The moment of pulse delivery to the controlling electrode is determined by the condition:

$$t_p > a_i \cdot t \& \leq a_i \cdot t + t_i$$

where t is the current time,  $a_i$  is the shift coefficient of the control pulse from the beginning of the EMF period.

Since the current time during simulation on the computer does not change continuously, but quantized with a step h, and the real controlling pulses have a finite duration, then it is necessary to verify the thyristor state and determine the value of its resistance Rvs at each integration step. Thyristor block-scheme of the thyristor state determination algorithm is given on the Figure 1. The Flag variable determines the state of the thyristor in the scheme. If Flag =1, the thyristor is opened, if Flag =0, the thyristor is closed.



**Figure 1.** Block-scheme of the thyristor state determination algorithm

The numerical forecast-correction method is used in paper [9]. However, this method has resulted in significant simulation time for long processes modeling in complex electrical circuits.

A method of accelerated calculation of transient processes in linear electrical circuits is proposed in papers [6-7].

In these papers, when transient processes are calculated, an equivalent circuit for special depictions is used, which is consisted of the following principles:

- ullet the integration time is not changed in steps, but in segments of length au, which contains several steps;
- in the equivalent circuit, the resistive element R has a resistance of  $R \cdot V$  and a source of constant EMF with

the value  $R \cdot i_0$  is turned on in series with it towards the current;

- inductive element L has resistance  $L \cdot D$ ;
- the capacitive element C=1/B has a resistance  $B \cdot S$  and a source of constant EMF with the value  $B\Delta i_{\theta} + u_{C\theta}$  is turned on in series with it towards the current,
  - inductive element L has resistance  $L \cdot D$ ;
- the capacitive element C=1/B has a resistance  $B \cdot S$  and a source of constant EMF with the value  $B\Delta i_0 + u_{C0}$  is turned on in series with it towards the current.

where **V**, **D**, **S** are special matrices that are calculated based on the use of orthogonal polynomials,  $\Delta$  is a vector containing the position of reference points on the segment,  $i_0$  is the initial value of the current in the resistive element,  $u_{C0}$  is the initial value of the voltage on the capacitive element.

Currents depiction  $C_k$ , for which Kirchhoff's laws are valid, appears in equivalent circuits for special depictions, instead of real currents  $i_k(t)$ . Depictions can be assumed like as currents. Depictions can be calculated on the basis of algebraic equations based on Kirchhoff's laws. The depiction of the currents  $C_k$  contains coefficient values of the current functions decomposition  $i_k(t)$  by orthogonal polynomials for the  $\kappa$ -th wire. Knowing the current decomposition coefficients  $i_k(t)$  by polynomials and the value  $i_{k0}$  for branch k, we can obtain a vector of current values  $I_k$  at all nodal points of the segment  $\tau$ .

For example, consider the circuit on the Figure 2.

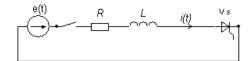


Figure 2. Circuit for simulation

The equivalent circuit for depictions is given on the Figure Figure. 3.

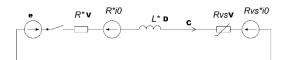


Figure 3. Equivalent circuits of the fig. 2. for depictions

The equation according to Kirchhoff's second law of the scheme on the Fig. 3 is:

$$(L\mathbf{D} + (R + Rvs)\mathbf{V}) \cdot \mathbf{C} = \mathbf{e} - (R + Rvs)i_0, \qquad (2)$$

or

$$\mathbf{Z} \cdot \mathbf{C} = \mathbf{F}, \tag{3}$$

where

$$\mathbf{Z} = L\mathbf{D} + (R + Rvs)\mathbf{V} , \qquad (4)$$

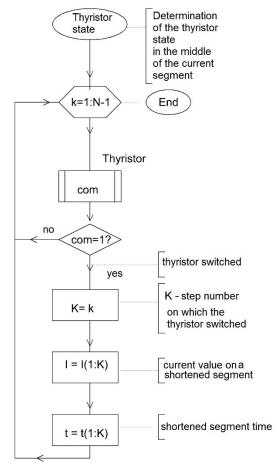
$$\mathbf{F} = \mathbf{e} - (R + Rvs)i_0. \tag{5}$$

The solution to equation (3) has the is:

$$\mathbf{C} = \mathbf{Z}^{-1} \cdot \mathbf{F} ,$$

where  $\mathbf{Z}^{-1}$  denotes the operation of taking the inverse matrix.

The error of equation solving (2) depends on the segment length of the simulation time. Therefore, the total interval of the simulation time should be divided into several segments.



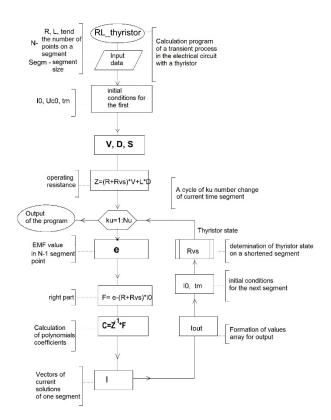
**Figure 4.** Block-scheme of the algorithm for determining the thyristor state on a shortened segment

The thyristor state can be changed in the middle of the segment. Therefore, the current segment must be shortened before the state changes. The block-scheme of the algorithm for determining the thyristor state on a shortened segment is given on the Figure 4.

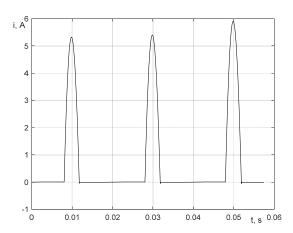
The block-scheme of the simulation algorithm of electric current change in the circuit of the Figure 1 is given on the Figure 5.

The computer program has been compiled on the basis of the developed theory in the octave-8.2.0-w64 system. Simulation results with input data are as follows:

Em=100 V; L=0.01 H; R=1 Ohm; Ti = 0.02 c; ti=0.01\*Ti; ai=0.4; f=50 is given on the Figure 6. A computer program based on Geer's numerical method has been also developed. The calculation results coincide with an accuracy of 0.01%, but the calculation time has increased more than 6 times.



**Figure 5.** Block-scheme of the simulation algorithm of electric current change in the circuit of the Figure 1



**Figure 6.** Time dependence of current obtained as a result of simulation

### **V. CONCLUSIONS**

The proposed method of transient processes calculation in electric circuits allows to replace operations with instantaneous values of currents into operations with constant currents in a special equivalent circuit. This approach allows reducing the calculation time significantly, what is important for the large schemes and long transient processes. In previous publications, this method has been successfully applied to continuous transients only. This paper shows the use of the proposed method in electric circuits that contain key elements thyristors.

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### СПЕКТРАЛЬНИЙ МЕТОД ПРИСКОРЕНОГО МОДЕЛЮВАННЯ ЕЛЕКТРИЧНИХ КІЛ ЩО МІСТЯТЬ ТИРИСТОРИ

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**Мета роботи.** Розробка методу розрахунку перехідних процесів в електричних колах з тиристорами на основі використання апроксимації функцій ортогональними поліномами.

**Методи дослідження.** Апроксимація функцій ортогональними поліномами, числові методи інтегрування диференціальних рівнянь, матричні методи, програмування, теорія електричних кіл.

**Отримані результати.** У роботі використаний метод поліноміальної апроксимації функції розв'язку інтегро-диференціальних рівнянь стану, які описує перехідні процеси електричного кола, що містить тиристори. використаний метод показав переваги перед іншими відомими методами у підвищенні точності та зниженні часу моделювання перехідних електричних процесів більш, ніж у 6 разів.

**Наукова новизна.** Рішення апроксимується рядом по поліномах Чебишова. інтегро-диференціальні рівняння стану перетворюються на лінійні алгебраїчні рівняння для спеціальних зображень функцій рішення. у схемі заміщення зображення функцій істинних струмів інтерпретуються як постійні струми. така схемна модель створює наочність досліднику, що виконує моделювання перехідних електричних процесів.

**Практична цінність.** Запропоновані методи відкривають можливість використання апарату теорії електричних кіл постійного струму для моделювання перехідних процесів у складних схемах, що містять тиристори.

Ключові слова: електричні кола; ортогональні поліноми; диференціальні рівняння; числові методи; спектральні методи; апроксимація; поліноми Чебишова; перехідні процеси; схемна модель